An Historical Overview of the Evolution of Productivity Analysis

by

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INTRODUCTION

Historically, productivity has been defined as an output-input ratio (e.g., Jorgenson and Griliches, 1967, REStud). In more recent times, it has also been defined as technical change (e.g., Diewert and Morrison, 1986, EJ), a combination of technical change and efficiency change (e.g., Fare et al, 1994, AER), or a residual (e.g., Olley and Pakes, 1996, E).

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Since the 1980s, it has been common to measure productivity change using quantity indices that do not satisfy common notions, or axioms, from measure theory. Today, economists are starting to compute measures of productivity change using 'proper' quantity indices.

Well-known economic models and statistical methods can be used to explain changes in proper measures of output and input change (and therefore productivity change). For example, stochastic frontier models can be used to decompose any proper productivity index into a combination of technical change, environmental change, various types of efficiency change, and a residual measure of changes in statistical noise.

Ahead in this talk . . .

- productivity = output/input
- productivity = technical change
- productivity = technical change $\times$ efficiency change
- productivity = a residual
- productivity analysis today

PRODUCTIVITY = OUTPUT/INPUT

Griliches (1996, JEL, p.1324) attributes the concept of an "output-over-input index" to Copeland (1937) and Copeland and Martin (1938).

"The story of productivity, the ratio of output to input, is at the heart of the record of man's efforts to raise himself from poverty" (Kendrick, 1961, p.3)

"... 'productivity' indexes measure those changes in output that have not been accounted for by the analyst's input measures" (Griliches, 1961, p.446)

"The rate of growth of total factor productivity is defined as the difference between the rate of growth of real product and the rate of growth of real factor input" (Jorgenson and Griliches, 1967, REStud, p. 250).

"[t]otal factor productivity ... is defined as output per unit of labor and capital combined" (Nadiri, 1970, JEL, p.1138)

"Productivity comparisons can be interpreted as comparisons of outputs relative to inputs" (Caves et al, 1982, EJ, p.79)

Productivity = Output/Input

"The conventional approach to the measurement of total factor productivity involves the computation of an index of total output and an index of all factor inputs. Total factor productivity is then simply computed as the ratio of the output index to the input index. Virtually all practitioners adopt this overall framework" (Christensen, 1975, AJAE, p.910)

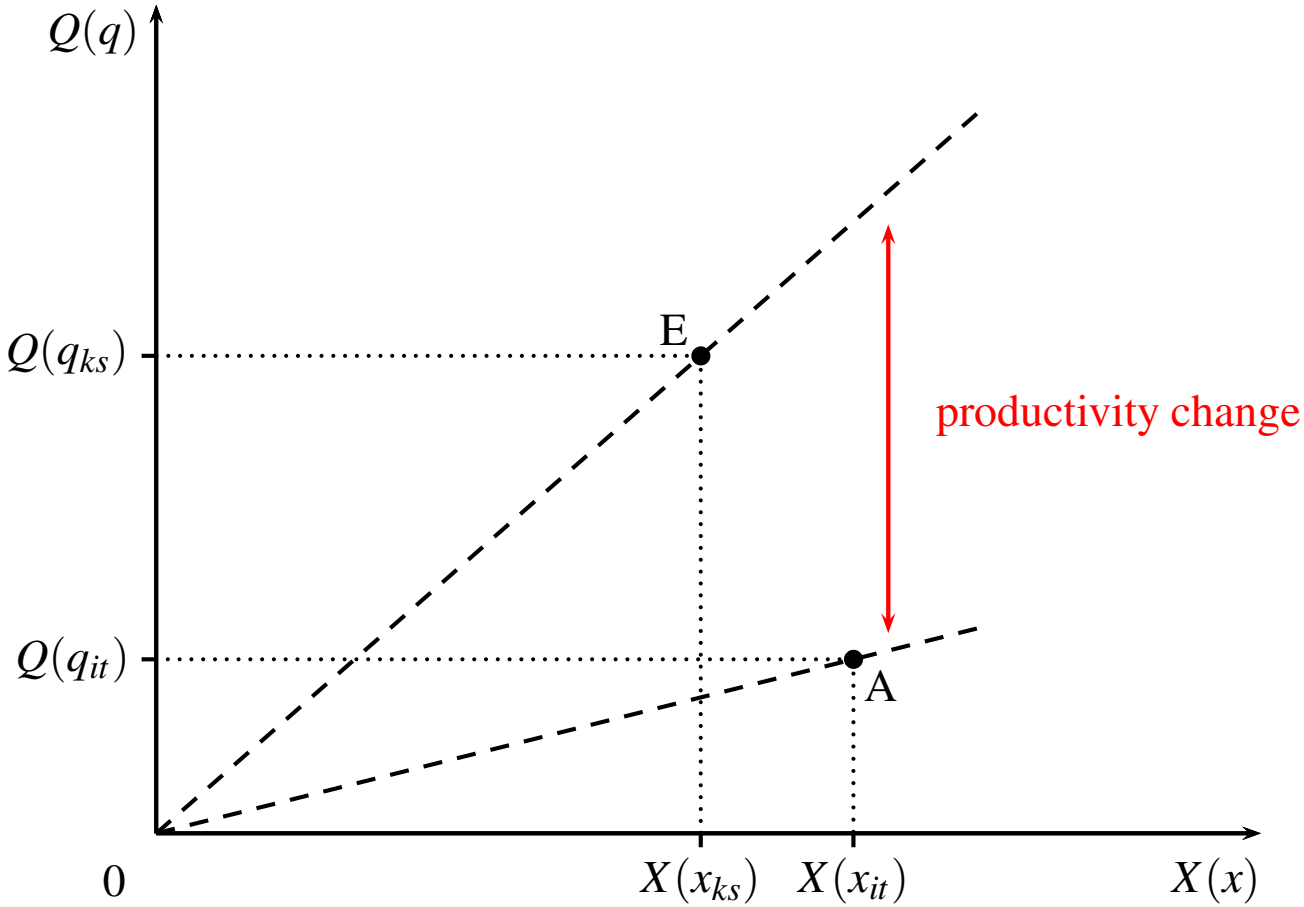
"Output per unit input, or TFP, is not a deeply theoretical concept." (Hulten, 2001, p.5).

"Productivity is commonly defined as a ratio of a volume measure of output to a volume measure of input use . . . there is no disagreement on this general notion" (Schreyer, 2001, OECD Manual, p.11)

"Simply put, productivity is efficiency in production: how much output is obtained from a given set of inputs. As such, it is typically expressed as an output-input ratio" (Syverson, 2011, JEL, p.329)

"Productivity is the ratio of output to input." (Jorgenson et al, 2011, JPA, p.162)

Productivity = Output/Input



Common Indices

It is common to measure output and input change (and therefore TFP change) using Fisher, Törnqvist and Malmquist indices. These are examples of binary indices. Binary indices do not generally satisfy a transitivity axiom. Thus, they should not be used to make multiple inter-temporal and/or inter-spatial comparisons.

Toy Example

Table 1. Common TFP Indices

Row	q	x_1	x_2	x_3	F	
A	1	1	1	1	1	
B	1	0.56	0.56	0.56	1.786	
C	2.37	1	1	1	2.37	
D	2.744	1.05	0.7	0.6	3.899	
E	2.744	1.05	0.7	0.6	4.081	
:	:	:	:	:	:	:
V	3.15	1	1	1	3.15	
W	2	0.919	0.919	0.919	2.176	
X	1	1.464	0.215	0.45	2.027	
Y	1	0.74	0.74	0.74	1.351	
Z	2.744	2.1	1.4	1.2	1.822	

F = Fisher

Toy Example

Table 1. Common TFP Indices

Row	q	x_1	x_2	x_3	F	T
A	1	1	1	1	1	1
B	1	0.56	0.56	0.56	1.786	1.786
C	2.37	1	1	1	2.37	2.37
D	2.744	1.05	0.7	0.6	3.899	3.879
E	2.744	1.05	0.7	0.6	4.081	4.088
:	:	:	:	:	:	:
V	3.15	1	1	1	3.15	3.15
W	2	0.919	0.919	0.919	2.176	2.176
X	1	1.464	0.215	0.45	2.027	1.936
Y	1	0.74	0.74	0.74	1.351	1.351
Z	2.744	2.1	1.4	1.2	1.822	1.807

F = Fisher; T = Tórnqvist

Common Indices

It is common to measure output and input change (and therefore TFP change) using Fisher, Törnqvist and Malmquist indices. These are examples of binary indices. Binary indices do not generally satisfy a transitivity axiom. Thus, they should not be used to make multiple inter-temporal and/or inter-spatial comparisons.

"[The] economics literature as well as the SNA93 are quite unanimous in this respect: for inter-temporal comparisons, changes over longer periods should be obtained by chaining: i.e., by linking the year-to-year-movements" (Schreyer, 2001, OECD Manual, p.83)

Toy Example

Table 1. Common TFP Indices

Row	q	x_1	x_2	x_3	F	T	CF	CT		
A	1	1	1	1	1	1	1	1		
B	1	0.56	0.56	0.56	1.786	1.786	1.786	1.786		
C	2.37	1	1	1	2.37	2.37	2.37	2.37		
D	2.744	1.05	0.7	0.6	3.899	3.879	3.903	3.883		
E	2.744	1.05	0.7	0.6	4.081	4.088	3.903	3.883		
:	:	:	:	:	:	:	:	:	:	:
V	3.15	1	1	1	3.15	3.15	3.680	4.274		
W	2	0.919	0.919	0.919	2.176	2.176	2.543	2.953		
X	1	1.464	0.215	0.45	2.027	1.936	2.025	2.378		
Y	1	0.74	0.74	0.74	1.351	1.351	1.480	1.777		
Z	2.744	2.1	1.4	1.2	1.822	1.807	1.968	2.351		

F = Fisher; T = Tórnqvist; CF = chained Fisher; CT = chained Törnqvist;

Common Indices

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"[The] economics literature as well as the SNA93 are quite unanimous in this respect: for inter-temporal comparisons, changes over longer periods should be obtained by chaining: i.e., by linking the year-to-year-movements" (Schreyer, 2001, OECD Manual, p.83)

"The superlative multilateral indexes that we have proposed are very attractive for cross section comparisons and for panel data comparisons, but they are not necessarily preferable to chain-linked bilateral indexes for time series comparisons." (Caves, Christensen, and Diewert, 1982, EJ, p.84)

Toy Example

Table 1. Common TFP Indices

Row	q	x_1	x_2	x_3	F	T	CF	CT	EKS	CCD
A	1	1	1	1	1	1	1	1	1	1
B	1	0.56	0.56	0.56	1.786	1.786	1.786	1.786	1.812	1.798
C	2.37	1	1	1	2.37	2.37	2.37	2.37	2.368	2.368
D	2.744	1.05	0.7	0.6	3.899	3.879	3.903	3.883	3.868	3.814
E	2.744	1.05	0.7	0.6	4.081	4.088	3.903	3.883	3.927	3.929
:	:	:	:	:	:	:	:	:	:	:
V	3.15	1	1	1	3.15	3.15	3.680	4.274	3.188	3.165
W	2	0.919	0.919	0.919	2.176	2.176	2.543	2.953	2.232	2.208
X	1	1.464	0.215	0.45	2.027	1.936	2.025	2.378	1.825	1.783
Y	1	0.74	0.74	0.74	1.351	1.351	1.480	1.777	1.363	1.355
Z	2.744	2.1	1.4	1.2	1.822	1.807	1.968	2.351	1.874	1.828

F = Fisher; T = Tórnqvist; CF = chained Fisher; CT = chained Törnqvist; EKS = Elteto-Koves-Szulc; CCD = Caves-Christensen-Diewert; some numbers may not add or multiply due to rounding.

Proper Indices

A proper index that compares the outputs of firm i in period t with the outputs of firm k in period s is any variable of the form $QI(q_{ks}, q_{it}) = Q(q_{it})/Q(q_{ks})$ where $Q(.)$ is a nonnegative, nondecreasing, linearly-homogeneous, scalar-valued aggregator function (O'Donnell, 2016, JEmet). A proper input index is defined in an analogous way.

A proper index that compares the TFP of firm i in period t with the TFP of firm k in period s is any proper output index divided by any proper input index (O'Donnell, 2016, JEmet).

These indices are proper in the sense that they satisfy basic axioms from measure theory (e.g., identity, transitivity, proportionality, homogeneity, time-space reversal, commensurability). The class of proper TFP indices includes the Lowe index defined by O'Donnell (2012, AJAE), the geometric Young (GY) index defined by O'Donnell (2016, JEmet), and the benefit of the doubt (BOD) index defined by O'Donnell (2018).

Toy Example

Table 2. Proper TFP Indices

Row	q	x_1	x_2	x_3	L	GY	BOD
A	1	1	1	1	1	1	1
B	1	0.56	0.56	0.56	1.786	1.786	1.786
C	2.37	1	1	1	2.37	2.37	2.37
D	2.744	1.05	0.7	0.6	3.903	3.766	3.519
E	2.744	1.05	0.7	0.6	3.903	3.766	3.519
:	:	:	:	:	:	:	:
V	3.15	1	1	1	3.15	3.15	3.15
W	2	0.919	0.919	0.919	2.176	2.176	2.176
X	1	1.464	0.215	0.45	2.084	2.29	3.906
Y	1	0.74	0.74	0.74	1.351	1.351	1.351
Z	2.744	2.1	1.4	1.2	1.951	1.883	1.759

L = Lowe; GY = geometric Young; BOD = benefit-of-the-doubt; some numbers may not add or multiply due to rounding.

Proper Indices

The choice of proper index is a matter of taste. "[W]e cannot hope for one ideal formula for the index number: if it works for the tastes of Jack Spratt, it won't work for his wife's tastes " (Samuelson and Swamy, 1974, AER).

PRODUCTIVITY = TECHNICAL CHANGE

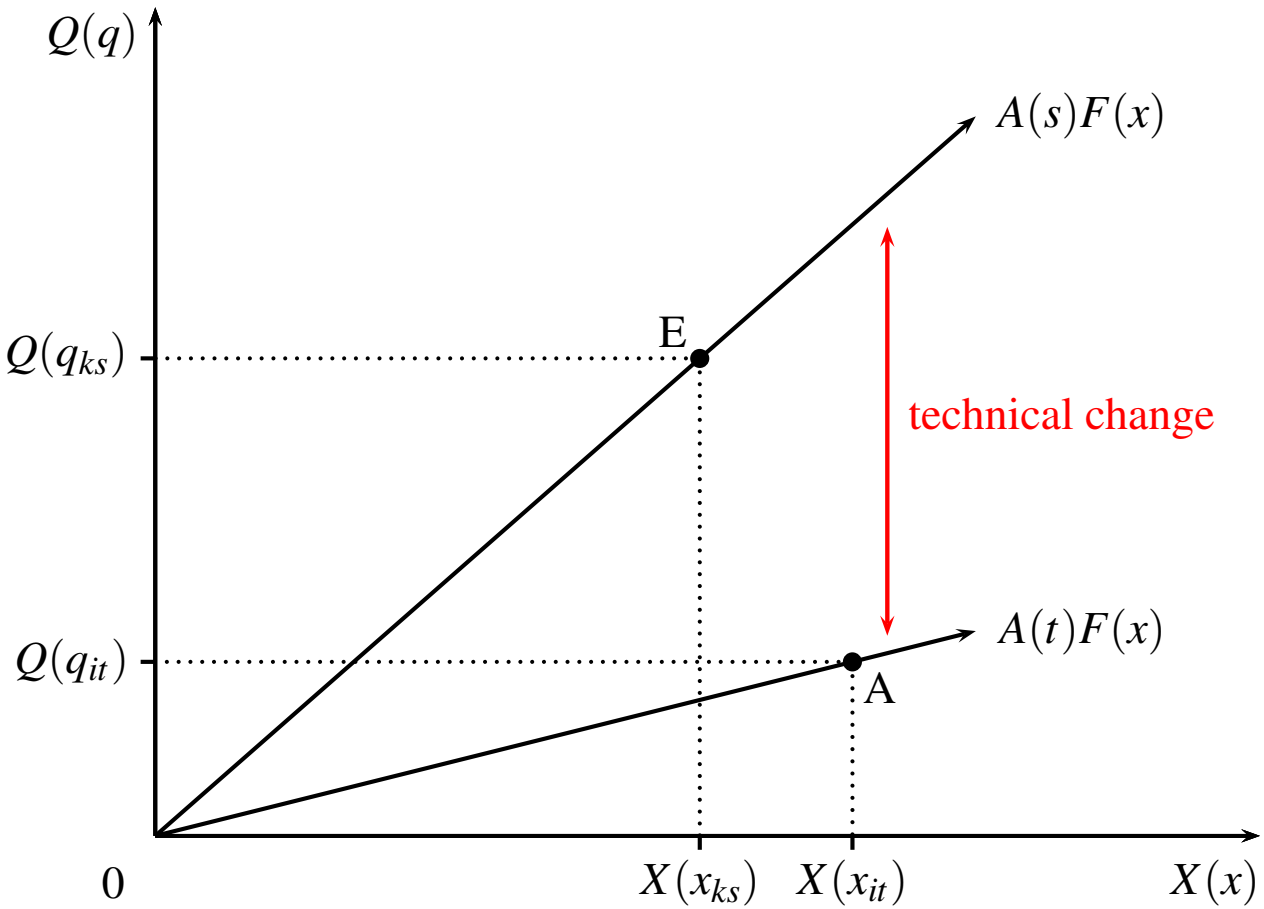
"I am using the phrase 'technical change' as a shorthand expression for *any kind of shift* in the production function" (Solow, 1957, REStat, p.312) (his italics)

"Shifts in the production function are identified with changes in total factor productivity" (Jorgenson and Griliches, 1967, REStud, p.249)

"...we address the problems related to measuring ...productivity growth or technical change" (Diewert and Morrison, 1986, EJ, p.659).

"[Welfare] depends on TFP (technical change) and on the effect of technical change on the cost of risk" (Orea and Wall, 2012, JAgE, p.103)

Productivity = Technical Change



Estimating Technical Change

When using time-series data, it is common to assume that

$$Q(q_t) = A(t)F(x_t) \tag{1}$$

where $Q(q_t)$ is an aggregate output, $A(t)$ is a measure of "technical change", and $F(.)$ is a linearly homogeneous, scalar-valued function (e.g., Solow, 1957, REStat). If $F(.)$ is also nonnegative and nondecreasing, then $A(t) = Q(q_t)/F(x_t)$ is a proper measure of TFP.

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There is more than one way of estimating $A(t)$. The approach used by Solow (1957, REStat) is underpinned by assumptions that are rarely, if ever, true. An alternative approach is to assume that $F(.)$ is a double-log function and rewrite (1) as

$$y_t = \alpha + \sum_{m=1}^{M} \beta_m \ln x_{mit} + e_t \tag{2}$$

where $y_t \equiv \ln Q(q_t)$ and $e_t \equiv \ln A(t)$. The unknown parameters can be estimated using least squares (LS) methods. Associated estimates of $A(t) = \exp(e_t)$ are given by the antilogarithms of the LS residuals.

Table 3. Estimates of Technical Change

Row	q	x_1	x_2	x_3	Solow	LS
A	1	1	1	1	1	1
B	1	0.56	0.56	0.56	1.440	1.786
C	2.37	1	1	1	1.795	2.37
D	2.744	1.05	0.7	0.6	2.458	4.200
E	2.744	1.05	0.7	0.6	2.458	4.200
:	:	:	:	:	:	:
V	3.15	1	1	1	0.344	3.15
W	2	0.919	0.919	0.919	0.190	2.176
X	1	1.464	0.215	0.45	0.123	2.804
Y	1	0.74	0.74	0.74	0.080	1.351
Z	2.744	2.1	1.4	1.2	0.094	2.100

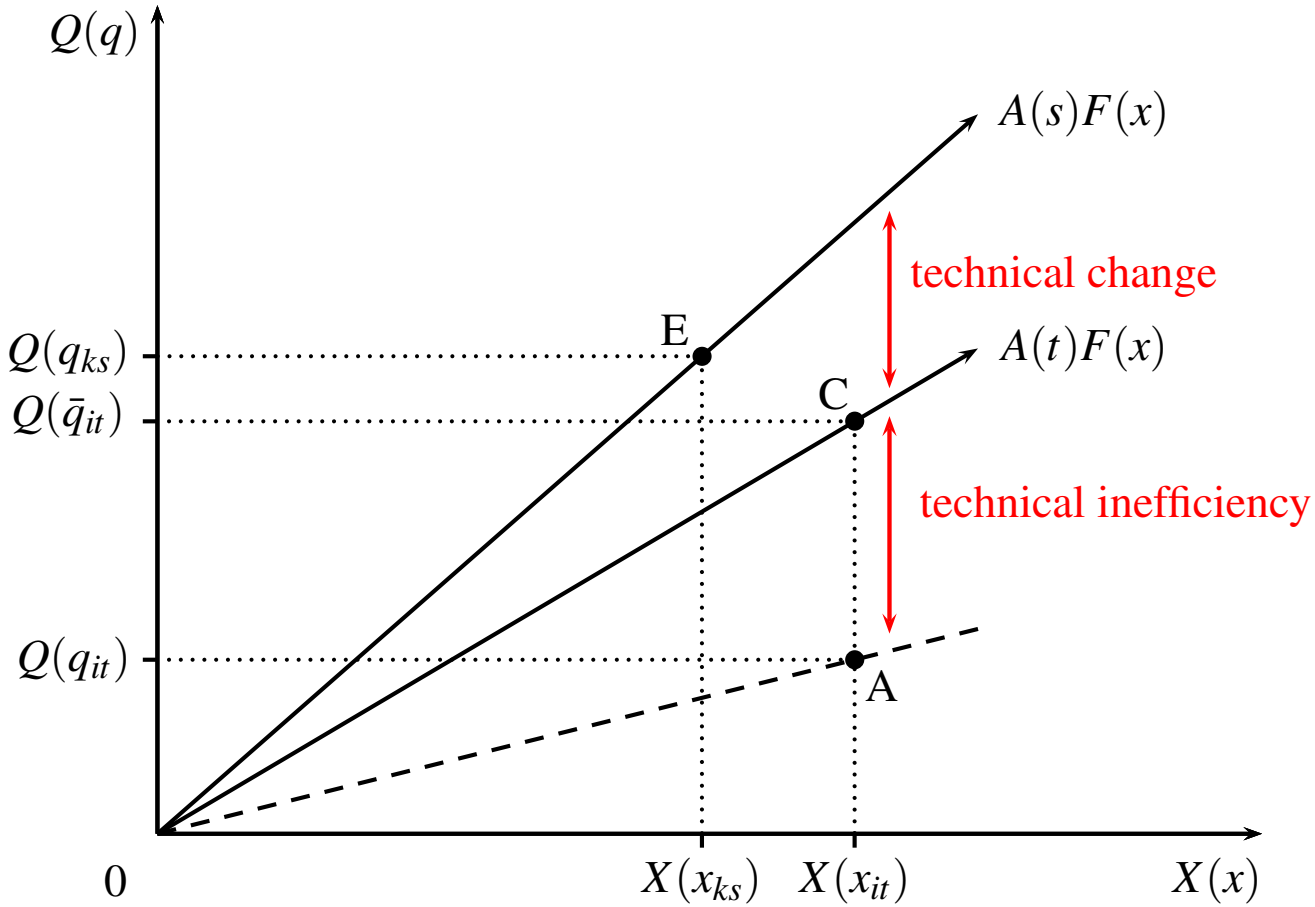
LS = least squares; some numbers may not add or multiply due to rounding.

PRODUCTIVITY = TECHNICAL CHANGE x EFFICIENCY CHANGE

"What is all important is total factor productivity (TFP) . . . a theory of TFP is needed. This theory must account for differences in TFP that arise for reasons other than the growth in the stock of technical knowledge" (Prescott, 1998, Int Econ Rev)

". . . we calculate productivity change as the geometric mean of two Malmquist productivity indexes . . . we define productivity growth as the product of [technical] efficiency change and technical change" (Färe et al, 1994, AER, pp.68,72)

Productivity = Technical Change x Technical Efficiency Change



Färe et al (1994, AER)

If the production frontier exhibits constant returns to scale (CRS), then the Malmquist productivity index (MPI) can be viewed as a binary measure of TFP change. It is common to make multiple inter-temporal comparisons of TFP by chaining MPI numbers.

Färe et al (1994, AER) estimate production frontiers and compute associated chained MPI numbers using data envelopment analysis (DEA). Their DEA approach is underpinned by the following assumptions:

- A1: all relevant quantities, prices and environmental variables are observed and measured without error;
- A2: production frontiers are locally (or piecewise) linear;
- A3: production frontiers exhibit CRS;
- A4: outputs, inputs and environmental variables are strongly disposable; and
- A5: production possibilities sets are convex.

The popularity of the approach derives partly from the availability of free software (e.g., DEAP; the `nonparaeff` package in R).

Toy Example

Table 4. Decomposing MPI Numbers Using DEA

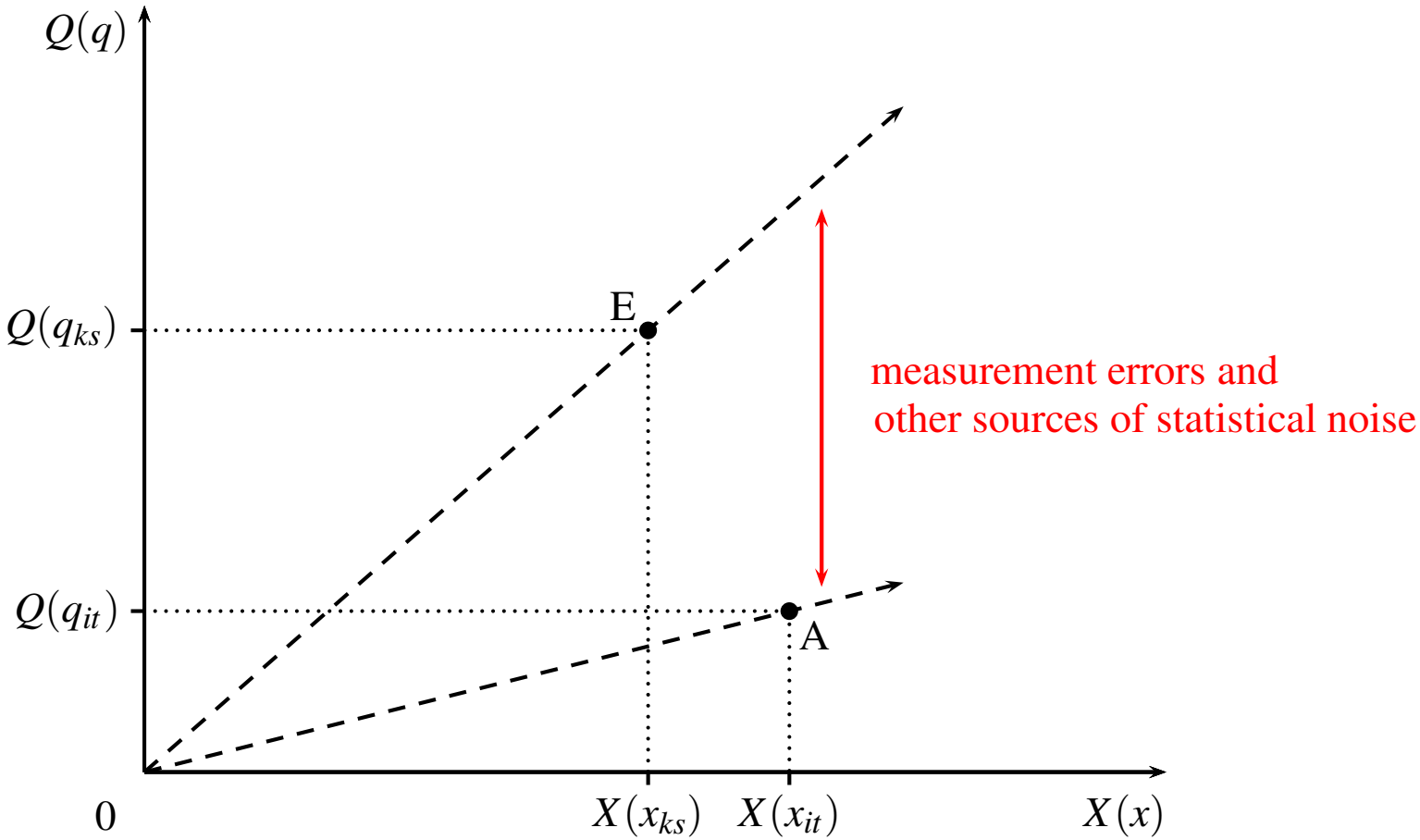
Row	q	x_1	x_2	x_3	MPI	= TC	× TEC
A	1	1	1	1	1	1	1
F	1	0.996	0.316	0.316	2.584	1.011	2.556
K	3.11	1	1	1	2.539	0.972	2.613
P	1	0.657	0.479	0.371	1.925	0.802	2.402
V	3.15	1	1	1	2.282	0.873	2.613

MPI = Malmquist productivity index; TC = technical change; TEC = technical efficiency change ; some numbers may not add or multiply due to rounding.

PRODUCTIVITY = A RESIDUAL

"Productivity is defined here, as elsewhere, as the residual from a relationship between deflated sales and inputs . . . If the industry has a single product and the deflator is specific to that product, then productivity has the traditional interpretation of a production function residual. If not, measured productivity is a residual from a reduced form sales equation . . ." (Olley and Pakes, fn.1, p.1264)

Productivity = A Residual



Olley and Pakes (1996, E) etc.

Equation (6) in Olley and Pakes (1996, E) is a "production function" of the form

$$q_{it} = A(a_{it})F(x_{it})\exp(\omega_{it} + \eta_{it})$$

where a_{it} is the age of the firm, ω_{it} represents productivity, and η_{it} is "either measurement error . . . or a shock to productivity" (p.1273). Subsequently, an estimate of $\exp(\omega_{it} + \eta_{it})$ is used as a measure of productivity (p.1287). Except in trivial cases, $\exp(\omega_{it} + \eta_{it})$ cannot be interpreted as an output-input ratio.

Olley and Pakes (1996, E) estimate the parameters in the production function as part of a simultaneous equations system. Other equations in the system are based on strong assumptions: (a) the prices of L and K are firm-invariant, (b) irrespective of how much L is used, a fixed proportion of the stock of K is consumed in each period, (c) at the beginning of each period, the manager knows how productive he/she will be in that period, but does not know how productive he/she will be in subsequent periods, and so on.

Similar assumptions have been made by Levinsohn and Petrin (2003, REStud) and Wooldridge (2009, EL). If the assumptions made by these authors are not true, then associated estimators and predictors will generally be biased.

Toy Example

Table 5. Residual Measures of Productivity Change

Row	q	x_1	x_2	x_3	OP	LP	W
A	1	1	1	1	1	1	1
B	1	0.56	0.56	0.56	1.329	1.669	0.380
C	2.37	1	1	1	2.37	2.37	2.37
D	2.744	1.05	0.7	0.6	3.172	4.037	0.839
E	2.744	1.05	0.7	0.6	3.172	4.037	0.839
:	:	:	:	:	:	:	:
V	3.15	1	1	1	3.15	3.15	3.15
W	2	0.919	0.919	0.919	2.085	2.155	1.737
X	1	1.464	0.215	0.45	1.844	2.748	0.204
Y	1	0.74	0.74	0.74	1.159	1.305	0.605
Z	2.744	2.1	1.4	1.2	2.258	2.189	2.668

OP = Olley-Pakes; LP = Levinsohn-Petrin; W = Wooldridge;
Note: Some numbers may not add or multiply due to rounding.

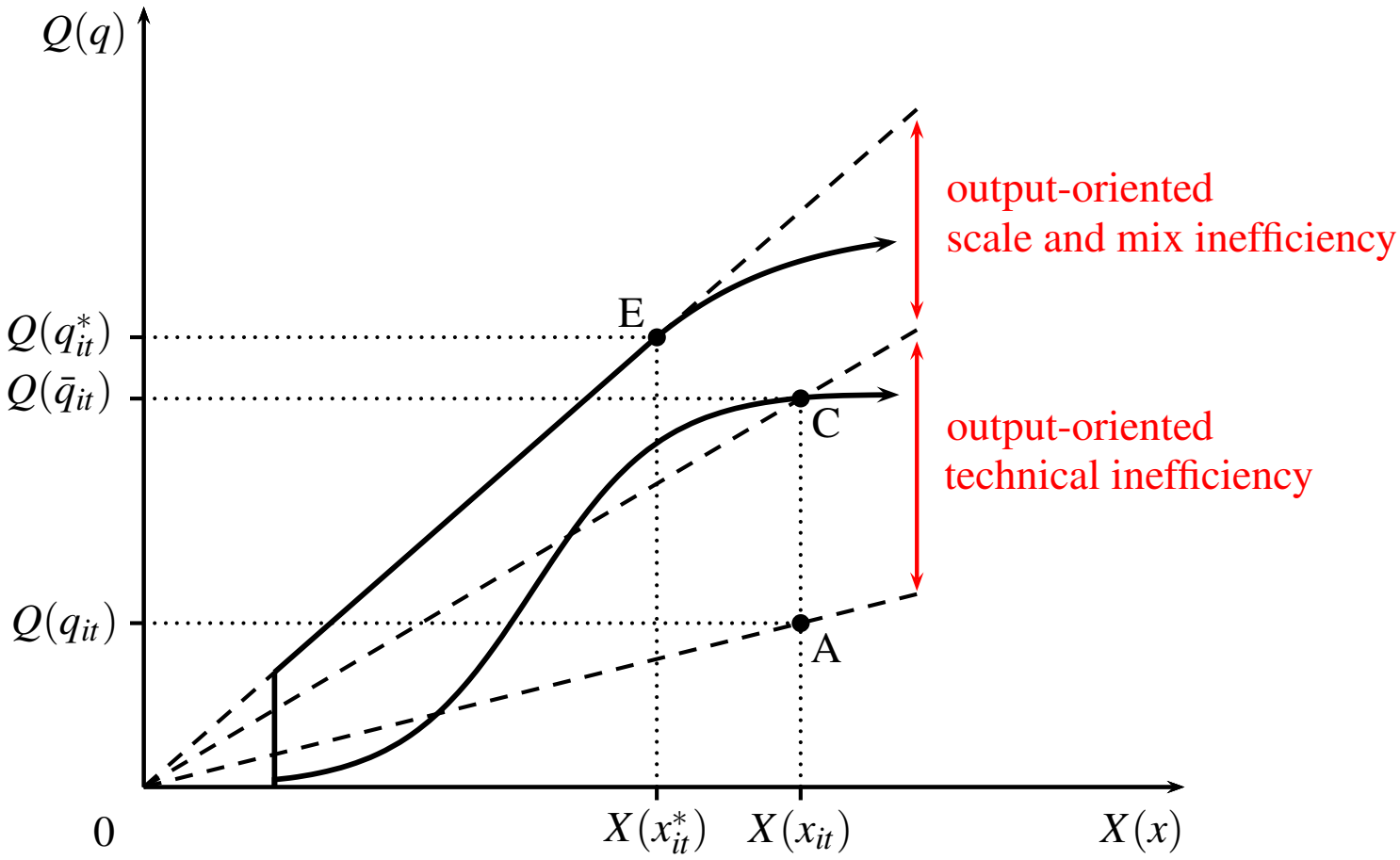
PRODUCTIVITY ANALYSIS TODAY

"...productivity is [still] measured as the ratio of an output quantity index to an input quantity index (or the difference between an output quantity indicator and an input quantity indicator)" (Grifell-Tatjé and Lovell, 2015, p.13)

In O'Donnell (2012, AJAE) uses DEA methods to decompose a Lowe TFP index into a measure of "technical change", a measure of scale and mix efficiency change, and a measure of technical efficiency change.

In O'Donnell (2016, JEmet) uses SFA methods to decompose a GY TFP index into a measure of technical change, a measure of environmental change, a measure of scale and mix efficiency change, a measure of technical efficiency change, and a residual measure of the change in statistical noise.

TFP(x, q) = TFP(x^*, q^*) × OSME × OTE



Adapted from Fig. 1 in O'Donnell (2016, JEmet).

The technical, scale and mix efficiency (TSME) of firm i in period t is $TSME^t(x_{it}, q_{it}, z_{it}) = TFP(x_{it}, q_{it})/TFP^t(z_{it})$ where $TFP(x_{it}, q_{it}) = Q(q_{it})/X(x_{it})$ is the TFP of the firm and $TFP^t(z_{it})$ is the maximum TFP possible in period t in an environment characterised by z_{it}.

The output-oriented technical efficiency (OTE) of firm i in period t is $OTE^t(x_{it}, q_{it}, z_{it}) = D_O^t(x_{it}, q_{it}, z_{it})$ where $D_O^t(.)$ is a period-and-environment--specific output distance function.

The output-oriented scale and mix efficiency (OSME) of firm i in period t is $OSME^t(x_{it}, q_{it}, z_{it}) = TSME^t(x_{it}, q_{it}, z_{it}) \ / OTE^t(x_{it}, q_{it}, z_{it})$.

After some simple algebra, any proper measure of TFP can be written as $TFP(x_{it}, q_{it}) = TFP^t(z_{it}) \times OSME^t(x_{it}, q_{it}, z_{it}) \times OTE^t(x_{it}, q_{it}, z_{it})$. This provides a basis for decomposing any proper TFP index into the product of an environment and technology index (ETI), an output-oriented scale and mix efficiency index (OSMEI), and an output-oriented technical efficiency index (OTEI). These components can be estimated using DEA.

Table 6. Decomposing Lowe TFPI Numbers Using DEA

Row	q	x_1	x_2	x_3	z	TFPI =	ETI ×	OSMEI ×	OTEI
A	1	1	1	1	1	1	1	1	1
B	1	0.56	0.56	0.56	1	1.786	1	0.696	2.566
C	2.37	1	1	1	1	2.37	1	1	2.37
D	2.744	1.05	0.7	0.6	1	3.903	1	1.521	2.566
E	2.744	1.05	0.7	0.6	1	3.903	1	1.521	2.566
:	:	:	:	:	:	:	:	:	:
V	3.15	1	1	1	1	3.15	1	1.261	2.498
W	2	0.919	0.919	0.919	2	2.176	1	1.272	1.711
X	1	1.464	0.215	0.45	2	2.084	1	0.812	2.566
Y	1	0.74	0.74	0.74	1	1.351	1	1.304	1.037
Z	2.744	2.1	1.4	1.2	1	1.951	1	0.897	2.176

TFPI = TFP index; ETI = environment and technology index; OSMEI = output-oriented scale and mix efficiency index; OTEI = output-oriented technical efficiency index; some numbers may not add or multiply due to rounding.

Equation (14) in O'Donnell (2016, JEmet) is a stochastic production frontier model with the following structure:

$$q_{1it} = A(t)Z(z_{it})f(x_{it}, q^*_{it}) \exp(-u_{it}) \exp(v_{it}) \tag{3}$$

where $A(t)$ is a measure of technical change, $Z(z_{it})$ is a measure of environmental change, $f(.)$ is a known function, $q^*_{it} = q_{it}/q_{1it}$ is a vector of normalised outputs, $u_{it} \equiv -\ln D^t_O(x_{it}, q_{it}, z_{it}) \geq 0$ represents output-oriented technical inefficiency, and v_{it} represents statistical noise.

After some simple algebra, any proper measure of TFP can be written as

$$Q(q_{it})/X(x_{it}) = A(t)Z(z_{it}) \left(\frac{f(x_{it}, q_{it})}{Q(q_{it})/X(x_{it})} \right) \exp(-u_{it}) \exp(v_{it}) \tag{4}$$

This provides a basis for decomposing any proper TFP index into the product of an output-oriented technology index (OTI), an output-oriented environment index (OEI), an OSMEI, an OTEI, and a statistical noise index (SNI). These components can be estimated using standard SFA methods.

Toy Example

Table 7. Decomposing Lowe TFPI Numbers Using SFA

Row	q	x_1	x_2	x_3	z	TFPI = OTI × OEI × OSMEI × OTEI × SNI				
A	1	1	1	1	1	1	1	1	1	1
B	1	0.56	0.56	0.56	1	1.786	1	1	1.132	1.577	1
C	2.37	1	1	1	1	2.37	1	1	1	2.37	1
D	2.744	1.05	0.7	0.6	1	3.903	1	1	1.094	3.569	1
E	2.744	1.05	0.7	0.6	1	3.903	1	1	1.094	3.569	1
:	:	:	:	:	:	:	:	:	:	:	:
V	3.15	1	1	1	1	3.15	1	1	1	3.15	1
W	2	0.919	0.919	0.919	2	2.176	1	0.799	1.018	2.674	1
X	1	1.464	0.215	0.45	2	2.084	1	0.799	1.064	2.451	1
Y	1	0.74	0.74	0.74	1	1.351	1	1	1.067	1.267	1
Z	2.744	2.1	1.4	1.2	1	1.951	1	1	0.943	2.070	1

TFPI = TFP index; OTI = output-oriented technology index; OEI = output-oriented environment index; OSMEI = output-oriented scale and mix efficiency index; OTEI = output-oriented technical efficiency index; SNI = statistical noise index; some numbers may not add or multiply due to rounding.

CONCLUSION

Historically, productivity has been defined as an output-input ratio. It is common to measure changes in outputs and inputs (and therefore output-input ratios) using indices that do not satisfy basic axioms from measure theory (e.g., identity, proportionality, transitivity). Analysis of these index numbers tells us nothing about changes in productivity (or, for that matter, anything else that might be of interest to economists, such as profits, revenues, costs and prices).

The first step in explaining productivity change is to compute proper measures of output and input change. Several such measures are now available. The next step is to use established models of firm behaviour (e.g., output maximisation, cost minimisation, expected profit maximisation) to help explain how outputs and inputs (and therefore output-input ratios) are determined. Economists are now starting to analyse productivity in this way.

Exact and Superlative Output Indices

If

(a) firms are price takers in output markets,

(b) firm managers maximise revenue,

(c) output prices and quantities are strictly positive,

(d) the period-t and period-s ODFs are nondecreasing in outputs, and

(e) the period-t and period-s ODFs are translog functions with identical second-order coefficients,

then GM index numbers are equal to Törnqvist index numbers (Caves et al, 1982a, Thm. 2). For this reason, the Törnqvist output index is said to be exact for a translog ODF.

Properties (d) and (e) cannot both be true. This means the Törnqvist index is merely superlative. Superlative output indices are exact for functions that can only provide approximations to ODFs.

Constant vs. Variable Weights

- "...constant weighting is not theoretically correct. If we compare 1913 with 1914, we need one set of weights; if we compare 1913 with 1915, we need, theoretically at least, another set of weights." (Fisher, 1922, p.275),
- "The fundamental difficulty is that, in most cases, particularly for geographical comparisons or comparisons between remote points of time, it is absurd to assume constant [weights]" (Frisch, 1936, Econometrica, p.6)
- "...along with Fisher and Frisch, we do not favor [constant weight] indexes" (Diewert and Fox, 2017, EJOR)

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- "...along with Fisher and Frisch, we do not favor [constant weight] indexes" (Diewert and Fox, 2017, EJOR)
- "...so enamoured did Fisher become with his so-called Ideal index ...that, when he discovered it failed the circular test, he had the hubris to declare '..., therefore, a *perfect* fulfilment of this so-called circular test should really be taken as proof that the formula which fulfils it is erroneous' (1922, p.271). Alas, Homer has nodded ..." (Samuelson and Swamy, l974, AER, p.575).

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- "Stock prices have reached what looks like a permanently high plateau." (Irving Fisher, New York Times, October 16, 1929. p.8.)

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- "Stock prices have reached what looks like a permanently high plateau." (Irving Fisher, New York Times, October 16, 1929. p.8.)
- "All the proof of a pudding is in the eating" (Camden, 1605)

Toy Example

Table 1. Common TFP Indices

Row	q	x_1	x_2	x_3	F	T	CF	CT	EKS	CCD
A	1	1	1	1	1	1	1	1	1	1
B	1	0.56	0.56	0.56	1.786	1.786	1.786	1.786	1.812	1.798
C	2.37	1	1	1	2.37	2.37	2.37	2.37	2.368	2.368
D	2.744	1.05	0.7	0.6	3.899	3.879	3.903	3.883	3.868	3.814
E	2.744	1.05	0.7	0.6	4.081	4.088	3.903	3.883	3.927	3.929
:	:	:	:	:	:	:	:	:	:	:
V	3.15	1	1	1	3.15	3.15	3.680	4.274	3.188	3.165
W	2	0.919	0.919	0.919	2.176	2.176	2.543	2.953	2.232	2.208
X	1	1.464	0.215	0.45	2.027	1.936	2.025	2.378	1.825	1.783
Y	1	0.74	0.74	0.74	1.351	1.351	1.480	1.777	1.363	1.355
Z	2.744	2.1	1.4	1.2	1.822	1.807	1.968	2.351	1.874	1.828

F = Fisher; T = Tórnqvist; CF = chained Fisher; CT = chained Törnqvist; EKS = Elteto-Koves-Szulc; CCD = Caves-Christensen-Diewert; some numbers may not add or multiply due to rounding.

Toy Example

Table 2. Proper TFP Indices

Row	q	x_1	x_2	x_3	L	GY	BOD
A	1	1	1	1	1	1	1
B	1	0.56	0.56	0.56	1.786	1.786	1.786
C	2.37	1	1	1	2.37	2.37	2.37
D	2.744	1.05	0.7	0.6	3.903	3.766	3.519
E	2.744	1.05	0.7	0.6	3.903	3.766	3.519
:	:	:	:	:	:	:	:
V	3.15	1	1	1	3.15	3.15	3.15
W	2	0.919	0.919	0.919	2.176	2.176	2.176
X	1	1.464	0.215	0.45	2.084	2.29	3.906
Y	1	0.74	0.74	0.74	1.351	1.351	1.351
Z	2.744	2.1	1.4	1.2	1.951	1.883	1.759

L = Lowe; GY = geometric Young; BOD = benefit-of-the-doubt; some numbers may not add or multiply due to rounding.

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